

**A Simplified Mathematical Model to Calculate the Maximum Usable  
Frequencies Over Iraqi Territory  
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**Abstract**

In this project, the spatial and temporal variation of maximum usable frequency (MUF) parameter is investigated and modeled. The values of MUF for Baghdad station links, with other stations distributed over Iraq territory, where determined using the international reference model (RECF533) for HF propagation. The determined MUF dataset reflected that the spatial distribution of this parameter for Baghdad station shows circular symmetry, which stimulated the possibility of using a two dimensional second order polynomial in order to describe MUF variation. The main feature of this adopted mathematical model is its low complexity. The results of conducted analysis indicated that the spatial variation of MUF is simple while its temporal variation is more complicated and needs more sophisticated mathematical description than the polynomial description. The test results showed that the accuracy of proposed simple model lies within the tolerance of REC533 reference model.

**Key words:** Ionosphere, HF communication prediction, MUF.

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## Introduction

The Maximum Usable Frequency, denoted by MUF, is an important ionospheric parameter. It is defined as the highest frequency that allows reliable long-range high frequency (HF) radio communication between two points due to ionospheric refraction. It is used to determine the workable frequencies for the long distance radio communications [1]. The long term observations showed that the variations in ionosphere have stochastic nature, and consequently it is difficult to assess the variation in all ionospheric parameter [2].

The literatures relevant to MUF parameter and its role in HF sky-wave radio communications are very extensive. The results of many studies conducted to describe the variations of maximum usable frequency have been published, and numerous programs, methods and models were developed. The complex calculations of the MUF parameter due to the complicity nature of the ionosphere layer let the road map to accurate modeling is still hard.

The revolutionary development in the computer world, occurred at 70's and 80's, caused drastic transition from the computer mainframe based methods to PC-based environment, and it leads to the birth of a class called microcomputer methods in the 1980s. Many of these methods still have validity in some applications. Since 1980's, several models and their programs have been built, such as MINIMUF [3], MICROMUF [4], Gerdes Approach [5], EINMUF [6], Devereux/ Wilkinson Method [7], HFBC84 [8], VOACAP [9], REC533 [10].

Most of the above mentioned models are proposed for global description of HF communication parameters, and they imply some complexity in their mathematical manipulation. In this paper a simple mathematical description is introduced to describe the MUF variation over Iraq territory.

## HF Communication

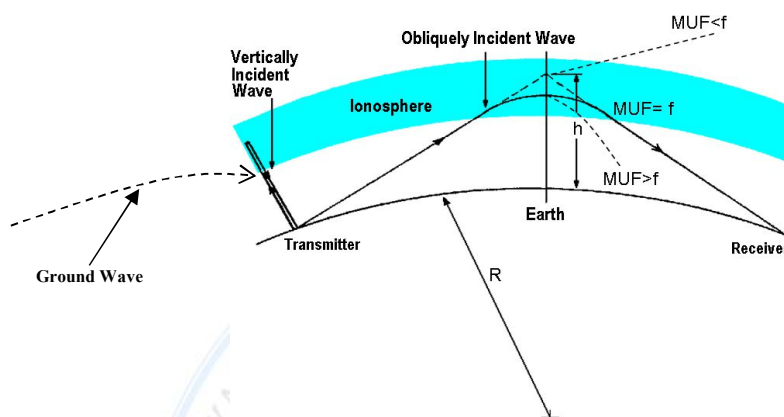
Ionosphere is created due to the ionization of the upper atmosphere by the electromagnetic radiation coming from the sun. Due to its ionization state it is gradually bends the HF radio waves coming from earth. Generally radio waves leave the transmitting antenna and hit the ionosphere obliquely.

The radio waves whose frequency is close the plasma (critical) frequencies are multi-refracted during their paths in ionosphere, and eventually they reflected back to earth. For any

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HF-communication link there is a maximum usable frequency (MUF), see figure (1). It depends on the state of ionosphere in the vicinity of refraction area. The frequencies higher than MUF can penetrate the ionosphere and travel far array in space.



**Fig. (1) The HF radio wave propagation**

The MUF depends mainly on the maximum electron density in F-region. Since, the electron density changes with the time of day, season, geographic location and solar activity [3], so MUF values show a sort of dependency upon the above parameters.

### REC533 Propagation Model

This propagation prediction model was developed to make a reliable and compatible estimation for HF-links (i.e., for radio frequencies lay within the range [3-30] MHz). At 90's, the International Telecommunication Union (ITU) made a series of recommendations about the methods of predicting the performance of High Frequency broadcast systems, among these recommendations is the ITU-R PI-533.

ITU considered this recommendation is so useful in the planning and operation of HF-transmission for the four seasons, hours of the day, geographic location and sunspot activities. ITU-R533 was derived from an old model, which was proposed in 1983 by CCIR Intern working party 6/12, and it was later refined by WARCS, CCIR and other organization.

At 1993, a PC/Windows program was made available to ITU by the working party 6A, (WP6A). Information on the availability of the program could be found in resolution-63 [11].

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This program is design to work in point to point mode, which means that the propagation for a path from a transmitter to a receiver is calculated. The program was designed to determine many HF-link parameters (like, monthly median basic MUF, Incident sky-wave field strength, and the available receiver power from lossless receiving antenna of given gain).

### MUF Distribution For Baghdad Station

In this project REC533 program have been used to stimulate the required MUF data set for Baghdad station HF-links for all months and day times. The generated data set was taken for the year 2001, because the solar activity in this year was in the peak of the solar activity cycle [12,13]. The median values for the 24 hours of the day and for 2 month of the year were determined.

The Baghdad station was considered as a transmitter station, and the other 26 stations, distributed over the provinces of Iraq territory were taken as receiving stations. Table (1) lists the locations of the selected stations, their distances, and the bearing angles relative to Baghdad station. Figure (2) shows the distribution of these stations over Iraq.

**Table (1) The list of stations, their distances, and bearing**

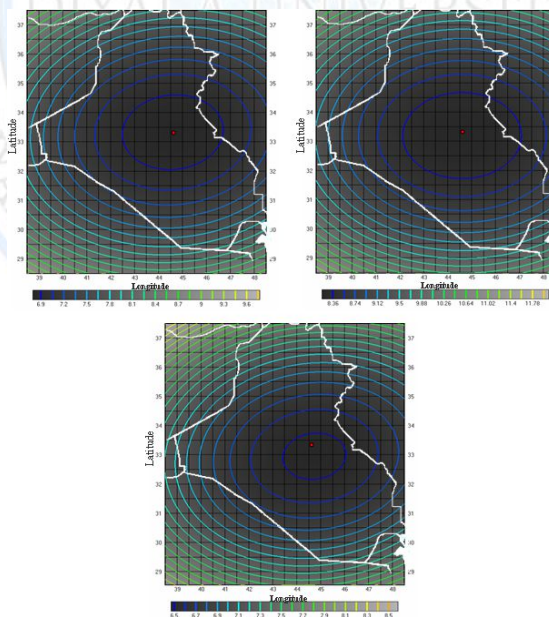
<i>Name</i>	<i>Longitude ( E )</i>	<i>Latitude ( N )</i>	<i>Distance ( Km )</i>	<i>Bearing (Deg.)</i>
Al-Musal	43.13	36.33	350.82	341.00
Sulaymaniyah	45.48	35.5	259.76	22.88
Mandely	45.60	33.73	120.84	69.51
Al-Basrah	47.78	30.50	3451.4	134.64
Al-Semawah	45.25	31.30	242.43	160.27
Al-Nekheab	42.27	32.15	238.46	235.93
Al-Rutbah	40.17	33.1	393.00	265.94
Al-Qa'im	42.05	34.37	243.47	297.80
Zakhow	42.70	37.30	465.40	340.86
Arbel	44.52	36.18	315.29	2.33
Karkuk	44.38	35.47	235.99	0.00
Sinjar	41.68	36.33	413.35	323.36



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Tikret	43.66	34.58	152.20	334.10
Al-Rumady	43.30	33.42	100.70	274.44
Ba'aquba	44.64	33.74	49.68	29.06
Khanaken	45.38	34.8	185.89	29.74
Karbala'a	44.03	32.60	89.67	201.38
Al-Kute	45.82	32.53	162.57	124.16
Al-Umara	47.17	31.86	309.75	122.37
Al-Dewania	44.94	31.99	160.23	160.88
Ajlan	44.27	30.04	368.61	181.62
Al-Nasereha	46.27	31.06	310.90	145.07
Al-Hather	43.72	35.58	255.52	346.29
Dewer	45.49	39.85	403.57	164.88
Heat	42.80	33.63	149.96	282.00
Rawandouz	44.55	36.61	363.22	2.45

The contour maps of MUF for different months and different time of the day indicated that there is a sort of circular symmetry in the MUF spatial distribution. Figure (3) shows samples of the contour maps.



**Fig. (3) The MUF contour taken for different times of the year.**

**Figures (4-6) presents the MUF variation for the different months and different receiver stations.**

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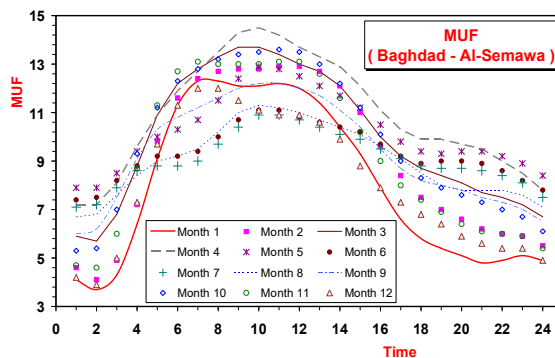


Fig. (5) MUF temporal variation for  
Baghdad-Rutba link

Figure (4) MUF temporal variation for  
Baghdad – Semawa link

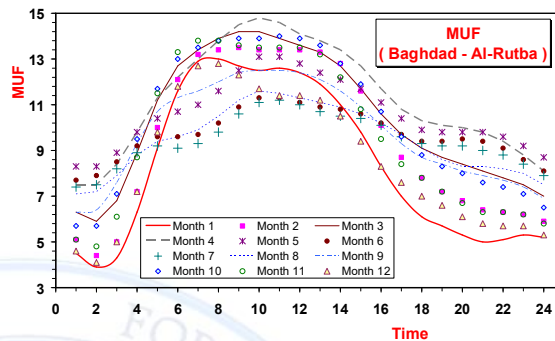
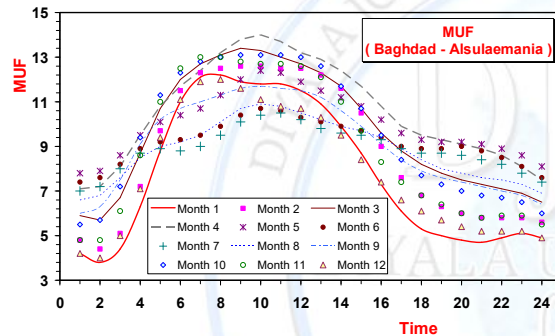


Fig. (6) MUF temporal variation for Baghdad –  
Sulaimani link



From above figures it is obvious that the variation of MUF median with daily time is more complicated than its monthly variation.

### The Proposed Simple MUF Model

Taking into consideration, the MUF variations shown in figure (1), and its symmetrical behavior (i.e., its directional invariance); therefore the spatial-temporal variation in MUF was approximated to be:

$$MUF = \sum_{n=0}^N a_n(m,t)d^n \quad (1)$$

Where, d is the distance between transmitter and receiver, m is the month number {1..12}, t is the local time [0..23], and n is the polynomial order. In this work a polynomial of order 3 was taken to represent MUF- distance relationship up to an acceptable level.

The polynomial coefficients {a's} depend on local time and month.

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As a first step in this research the set of a's coefficients for 12 months and 24 hours was determined. Table (2) shows sample of a's coefficients for July-2001.

**Table (2) A sample of the a's coefficients for July 2001**

<i>T</i>	<i>a<sub>0</sub></i>	<i>a<sub>1</sub></i>	<i>a<sub>2</sub></i>	<i>a<sub>3</sub></i>
0	6.233	9.32E-03	-3.70E-05	5.15E-08
1	6.503	8.71E-03	-3.78E-05	5.63E-08
2	6.533	1.64E-02	-6.65E-05	9.06E-08
3	7.446	1.32E-02	-5.33E-05	7.44E-08
4	7.876	1.01E-02	-4.06E-05	5.96E-08
5	7.817	1.10E-02	-4.53E-05	6.46E-08
6	7.816	1.40E-02	-5.83E-05	8.21E-08
7	7.575	2.63E-02	-1.11E-04	1.47E-07
8	7.867	3.06E-02	-1.27E-04	1.68E-07
9	7.765	3.79E-02	-1.57E-04	2.06E-07
10	7.905	3.53E-02	-1.43E-04	1.86E-07
11	7.333	3.89E-02	-1.55E-04	1.99E-07
12	7.003	3.97E-02	-1.60E-04	2.06E-07
13	7.048	3.64E-02	-1.48E-04	1.93E-07
14	7.321	2.93E-02	-1.17E-04	1.54E-07
15	7.833	1.72E-02	-6.46E-05	8.63E-08
16	8.403	5.16E-03	-1.80E-05	2.93E-08
17	8.485	1.20E-03	-1.94E-06	7.87E-09
18	8.350	1.31E-03	-5.96E-07	5.56E-09
19	8.641	-2.28E-03	1.09E-05	-6.35E-09
20	7.875	5.37E-03	-1.77E-05	2.66E-08
21	7.326	1.06E-02	-4.02E-05	5.49E-08
22	6.766	1.45E-02	-5.79E-05	7.74E-08
23	6.649	9.25E-03	-3.75E-05	5.29E-08

As second step the coefficients a's were represented as two dimensional polynomial with respect to local time and day number, such that:

$$a(m, t) = \sum_{k=0}^K b_k(t) m^k \quad (2)$$

$$b(t) = \sum_{j=0}^J c_j t^j \quad (3)$$

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The polynomial order (K) for t is taken 4, and for m is taken 3. For determining the values of c coefficients the least mean square error method was applied, where the error is:

$$\chi^2 = \sum_{i=0}^L \left( MUF(i) - \sum_{n=0}^N d^n \left[ \sum_{k=0}^K \left( \sum_{j=0}^J c_j t^j \right) m^k \right] \right)^2 \quad (4)$$

$$\frac{\partial \chi^2}{\partial c_j} = 0 \quad \forall j \quad (5)$$

Where, L is the number of MUF data set. For computing values of {c} the used data set was for 26 HF-links, 12 months and 24 hours.

Table (3) lists the determined values of the coefficients  $c_j$  for all months and local time, the overall determined MSE was found (0.56).

**Table (3) The determined values of the (c)  
coefficients**

	$C_0$	$C_1$	$C_2$	$C_3$
1	-1.8E-07	9.1E-08	-1.1E-08	3.5E-10
2	-1.2E-07	7.3E-08	-8.9E-09	2.9E-10
3	6.6E-08	6.4E-09	-9.4E-10	6.6E-11
4	2.3E-07	-1.9E-08	-2.9E-09	4.1E-10
5	3.7E-07	-8.9E-08	5.8E-09	1.1E-10
6	2.5E-07	-3.6E-08	-1.3E-09	3.7E-10
7	1.3E-07	1.7E-08	-5.4E-09	3.4E-10
8	4.0E-08	5.9E-08	-7.9E-09	2.7E-10
9	4.7E-08	6.1E-08	-6.5E-09	1.3E-10
10	1.6E-07	-8.7E-10	4.4E-09	-4.0E-10
11	2.1E-07	-2.9E-08	7.6E-09	-4.8E-10
12	2.5E-07	-4.5E-08	1.0E-08	-5.8E-10
13	1.9E-07	-5.3E-09	2.7E-09	-2.1E-10
14	1.7E-07	-3.1E-09	2.2E-09	-1.8E-10
15	2.1E-07	-3.1E-08	4.8E-09	-2.3E-10
16	2.4E-07	-4.6E-08	3.5E-09	2.2E-12
17	2.0E-07	-2.6E-08	-2.2E-09	3.8E-10
18	1.7E-07	-6.2E-09	-6.3E-09	5.9E-10
19	1.4E-07	2.3E-08	-1.1E-08	7.9E-10
20	8.1E-08	4.7E-08	-1.3E-08	8.0E-10
21	1.7E-08	7.3E-08	-1.6E-08	8.9E-10
22	-3.4E-08	8.1E-08	-1.4E-08	7.0E-10
23	-5.4E-08	7.8E-08	-1.3E-08	5.8E-10
24	-1.4E-07	9.2E-08	-1.3E-08	4.8E-10



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### Conclusions

1. The spatial distribution of MUF values is semi-circular, which let the possibility of using 2<sup>nd</sup> order polynomial is acceptable.
2. The accuracy of representing the time variation using mathematical expression that have slow variation scale (like polynomial) is less accurate than using same representations for spatial variations.

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