



Study the Electric Quadrupole Transition (E2) in  
Sr<sub>38</sub> and Zr<sub>40</sub> Nuclei

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**Abstract**

Transition strengths  $[M(E2; 2^+ \rightarrow 0^+)]^2_{w.u.}$  produced by pure electric quadrupole emission in even-even isotopes of Sr and Zr have been calculated and plotted as a function for neutron number (N) by using life times of  $2^+_1$  excited states and the  $\gamma_0$  intensities. Similar  $[M(E2)]^2_{w.u.}$  curves with stable valley ranging between neutron number 48 to 58 have been obtained for Sr and Zr with proton sub-shell closed at 38, 40 respectively. For sake for comparison Transition strengths  $[M(E2; 2^+ \rightarrow 0^+)]^2_{w.u.}$  are converted to reduced transition probabilities  $B(E2; 2^+ \rightarrow 0^+)$  for <sup>38</sup>Sr, <sup>40</sup>Zr nuclides to comparison with other results.

**Keywords:** Electric Quadrupole Transition (E2), <sup>38</sup>Sr and <sup>40</sup>Zr Nuclei, reduced transition probabilities.

دراسة الانتقالات لرباعي القطب الكهربائي (E2) في نويدات Sr<sub>38</sub> و Zr<sub>40</sub>

**الخلاصة :**

تم حساب قوى الانتقال  $[M(E2; 2^+ \rightarrow 0^+)]^2_{w.u.}$  لانقالات كما الناتجة عن اشعاع رباعي قطب كهربائي نقي للنظائر الزوجية زوجية لكل من <sup>38</sup>Sr و <sup>40</sup>Zr كدالة إلى العدد النيوتروني. حيث حسبت قوى الانتقال  $[M(E2; 2^+ \rightarrow 0^+)]^2_{w.u.}$  بالاعتماد على معدل العمر للمستوي المتهيج الأول  $2^+_1$  والشدة النسبية لأشعة كما المنبعثة من ذلك المستوي المحفز إلى المستوي الأرضي أن العلاقة بين قوى الانتقال وبين العدد النيوتروني تكون على شكل منحنى له أقل قيمة لقوى الانتقال  $[M(E2; 2^+ \rightarrow 0^+)]^2_{w.u.}$  في مدى العدد النيوتروني 48-58 لكل من Sr و Zr وقد لوحظ أن شكلي المنحنين المتشابه يعود إلى أن كل من Sr و Zr له المستوي الثانوي المغلق عند العدد الذري 40, 38 على التوالي. ولغرض المقارنة فقد تم تحويل قيم قوى الانتقال  $[M(E2; 2^+ \rightarrow 0^+)]^2_{w.u.}$  إلى احتمالية الانتقال المختزلة  $B(E2; 2^+ \rightarrow 0^+)$  لتلك الانتقالات. إن عملنا الحالي يعطي مجموعة كاملة لاحتمالية الانتقال المختزلة في النويدات المذكورة لغرض المقارنة مع نتائج تم حسابها سابقاً.



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**Introduction**

Electric quadrupole transitions strengths  $[M(E2)]^2_{W.u.}$  measurements in even-even nuclei are provided good information about nuclear structure. The great amount of measured  $[M(E2)]^2_{W.u.}$  values versus neutrons number in <sup>38</sup>Sr and <sup>40</sup>Zr give systematic curves with stable valleys which serve to determine the relative importance of the collective and single-particle effects to describe the <sup>38</sup>Sr and <sup>40</sup>Zr structures.

The Weisskopf single-particle transition probability  $B(EL, ML)$  is defined by [1] as the ratio of the single-particle half-life time to the experimental half-life time for gamma transition

$$B(EL, ML)_{W.u.} \downarrow = \frac{t_{1/2}^{\gamma}(EL, ML)_{sp}}{t_{1/2}^{\gamma}(EL, ML)_{exp}} \dots \dots \dots (1)$$

Where L is the multipolarities  $L=1, 2, 3, \dots$   
 $L \neq 0$

While the  $\gamma$ -ray transition strength  $[M(EL, ML)]^2$  is defined as the ratio of gamma width to gamma width in Weisskopf unit (W.u.) [2]

$$[M(EL, ML)]^2_{W.u.} \downarrow = \frac{\Gamma(EL, ML)_{exp}}{\Gamma(EL, ML)_{W.u.}} \dots \dots \dots (2)$$

Since  $\Gamma_{\gamma} T \approx \hbar$  ..... (3)

Where;

$\Gamma_{\gamma}$  is the total width

$$\Gamma_{\gamma} = \sum \Gamma_{\gamma i} \dots \dots \dots (4)$$

$\Gamma_{\gamma i}$  is the partial gamma width

T is the mean life time of initial level

$$T = \frac{\tau_{1/2}}{\ln 2} \dots \dots \dots (5)$$

$\hbar = \frac{h}{2\pi} = 0.65822 \times 10^{-15} \text{ eV.s}$ , h is Plank constant.  
From eqs. (2, 3 and 4) . can be concluded

$$B(EL, ML)_{W.u.} \downarrow = [M(EL, ML)]^2_{W.u.} \dots \dots \dots (6)$$

Specific expression for  $B(EL, ML)_{W.u.}$  suggested [3] is :

$$B(EL, ML)_{W.u.} \downarrow = \dots \dots \dots (7)$$

If the transition is of mixed multi polarity M1 and E2 ref.[4] then

$$\delta = \pm \dots \dots \dots (8)$$

Where  $\delta$  is the mixing ratio

$$\text{and } \Gamma_{\gamma} = \Gamma(M1) + \Gamma(E2) \dots \dots \dots (9)$$

For a pure E2 transition,  $\delta=0$  and hence



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$$\Gamma(E2) = \Gamma_{\gamma} \dots\dots\dots(10)$$

Then the transition strength for electric quadrupole transition E2 can be calculated by using eq (2) in the form :

$$[M(E2)]^2_{w.u.\downarrow} = \dots\dots\dots(11)$$

Or eq (7) in the form :

$$[M(E2)]^2_{w.u.\downarrow} = \dots\dots\dots(12)$$

On the basis of an extreme single particle model the value for the  $\Gamma(E2)_{w.u.}$  in eV. [3]

$$\Gamma(E2)_{w.u.} = 4.7907 \times 10^{-23} A^{4/3} E_{\gamma}^5 \dots\dots(13)$$

Where E in KeV. for nuclear of mass No. A

and the corresponding reduced transition probability is :

$$B_{w.u.}(E2) = 0.05940 e^2 (fm)^4 \dots\dots(14)$$

The relation between  $B(E2)_{\downarrow} = B(E2; 2 \rightarrow 1)$  and  $B(E2)_{\uparrow} = B(E2; 1 \rightarrow 2)$  as given by ref [3] is:

$$B(E2)_{\uparrow} = B(E2)_{\downarrow} \dots\dots\dots(15)$$

### Results of Calculations

The electric quadrupole transition strengths for the  $2^+ \rightarrow 0^+_{g.s.}$  transition have been calculated as a function of neutron number (N) using eq. (11) with aid of the experimental data reported in ref. [1] to even-even isotopes for; <sup>38</sup>Sr ( $74 \leq A \leq 104$ ) and <sup>40</sup>Zr ( $80 \leq A \leq 110$ ) which have only one transition for  $\gamma$  is  $\gamma_0$  with intensity (100%)E2.

The results of calculations are presented in table (1) for <sup>38</sup>Sr nuclides and in table (2) for <sup>40</sup>Zr nuclides. . The transition strengths are plotted as a function of neutron number (N) as shown in Fig. (1) and Fig. (2) for <sup>38</sup>Sr and <sup>40</sup>Zr respectively . For the sake of comparison, the values are converted to  $B(E2) e^2 b^2_{\uparrow}$  using eq. (12) and then eq.(15), the present  $B(E2) e^2 b^2_{\uparrow}$  values of  $\gamma_0$  -transitions in <sup>38</sup>Sr and <sup>40</sup>Zr nuclides are compared with the experimental values as well as with other of various theoretical models such as; Single Shell Asymptotic Nilsson Model (SSANM) and Finite-Range Droplet Model(FRDM) reported in ref.[5]. This comparison are presented in tables (3&4) and shown in Figs. (3 &4) respectively.



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Table (1): Transition strengths of - transitions from with total gamma widths and mean life time for the first excited states of <sup>38</sup>Sr . The experimental data of ref.[1] are used in the present calculations.

Experimental data of ref.[1]				(ps)	(eV)	
N	(keV)	{keV}	t <sub>1/2</sub> (ps)			
40	278.5	178.5	155±19	223.6652±27.41 70	2492(360)	109.98±13.45
42	385.86	385.86	35±1.3	50.5050±1.8759	13032(484)	92.26±3.42
44	573.54	573.64	8.9±0.4	12.8427±0.5772	51252(2303)	48.35±2.17
46	793.30	793.3				
48	1076.68	1076.64	1.61±0.06	2.3232±0.0865	283324(10549)	10.77±0.40
50	1836.08 7	1836.063	0.162±0.00 5	0.2337±0.0072	2816516(86773 )	7.20±0.22
52	831.68	831.69	7±2	10.1010±2.8860	65163(18618)	8.47±2.42
54	814.98	814.98	8±3	11.5440±4.3290	57018(21381)	7.97±2.99
56	836.91	836.9	6.9±2.8	9.9567±4.0404	66108(26826)	7.86±3.19
58	814.93	815	4.8±2.8	6.9264±4.0404	95030(55434)	12.55±7.32
60	144.225	144.224	2800±80	4040.4±115.4	162.909(4652)	120.62±3.44
62	129.7	129.7	3910±160	5642.1±230.8	116.622(4772)	142.90±5.85
64	126	126	3000±1200	4329±1731.6	152.048(61030)	20.97±8.42



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Table (2): Transition strengths of  $\gamma$ -transitions from with total gamma widths, mean life time for the first excited states of  $^{40}\text{Zr}$ . The experimental data of ref.[1] are used in the calculations.

Experimental data of ref.[1]				(ps)	(eV)	
N	(keV)	(keV)	$t_{1/2}$ (ps)			
42	407.30	407.3	$28 \pm 3$	$40.4040 \pm 4.3290$	16291(1745)	$85.16 \pm 9.12$
44	540	540	$14.1 \pm 0.8$	$20.3463 \pm 1.1544$	32350(1835)	$39.98 \pm 2.27$
46	751.75	751.74	$7.3 \pm 1.4$	$10.5339 \pm 2.0202$	62485(11983)	$14.31 \pm 2.74$
48	1057.03	1057	$0.8 \pm 0.3$	$1.1544 \pm 0.4329$	570183(213818)	$23.04 \pm 8.64$
50	2186.27 4	2186.242	—	$0.205 \pm 0.01^*$	3211854.8(1973555)	$3.33 \pm 0.20$
52	934.49	934.49	$5 \pm 0.4$	$7.215 \pm 0.5772$	91229(7298)	$6.43 \pm 0.51$
54	918.75	918.74	$7.7 \pm 0.8$	$11.1111 \pm 1.1544$	5923(6407)	$4.42 \pm 0.48$
56	1750.49 8	1750.42				
58	1222.93	1222	$< 200$	$< 288.6$	$> 2244.6944(1710547)$	$> 0.04 \pm 0.002$
60	212.530	212.531	$540 \pm 20$	$779.2 \pm 28.8$	844.738(31222)	$87.61 \pm 3.24$
62	151.78	151.75	$1910 \pm 250$	$2756.1 \pm 360.7$	238.822(31255)	$129.90 \pm 17$

\*mean life time for the transition was taken from ref.[5]

Figure (1): Transition strengths, for  $\gamma$ -transitions as a function of the neutron number in  $^{38}\text{Sr}$  nuclei.

Figure (2): Transition strengths, for  $\gamma$ -transitions as a function of the neutron number in  $^{40}\text{Zr}$  nuclei.



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Table (3): The calculated reduced transition probabilities B (E2)  $e^2b^2 \uparrow$  values are compared with that of experimental and, theoretical predications for <sup>38</sup>Sr nuclides .

A	N	(keV)	(keV)	B(E2;) $e^2b^2 \uparrow$			
				Experimental values of ref.[5]	Theoretical values		
					Present work	SSANM of ref.[5]	FRDM of ref[5]
78	40	278.5	178.5	1.08±0.15	1.081±0.133	0.732	1.264
80	42	385.86	385.86	0.959±0.036	0.946±0.035	0.661	0.017
82	44	573.54	573.64	0.513±0.020	0.512±0.023	0.554	0.018
84	46	793.30	793.3	0.289±0.044	.170	0.452	0.018
86	48	1076.68	1076.64	0.128±0.014	0.121±0.004	0.362	0.019
88	50	1836.08 7	1836.06 3	0.092±0.005	0.083±0.002	0.247	0.014
90	52	831.68	831.69	0.113±0.034	0.101±0.029	0.469	0.020
92	54	814.98	814.98	0.114±0.048	0.098±0.036	0.606	0.048
94	56	836.91	836.9	0.118±0.047	0.099±0.040	0.743	0.566
96	58	814.93	815	0.24±0.14	0.164±0.095	0.874	1.159
98	60	144.225	144.224	1.282±0.039	1.621±0.046	0.985	1.382
100	62	129.7	129.7	1.42±0.08	1.974±0.080	1.047	1.457
102	64	126	126	—	0.297±0.119	1.045	1.441



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A	N	(keV)	(keV)	B(E2;) e <sup>2</sup> b <sup>2</sup> ↑			
				Experimental values of ref.[5]	Theoretical values		
					Present work	SSANM of ref.[5]	FRDM of ref.[5]
82	42	407.30	407.3	0.91±0.09	0.902±0.096	0.740	0.020
84	44	540	540	0.438±0.025	0.437±0.024	0.622	0.020
86	46	751.75	751.74	0.166±0.031	0.161±0.030	0.510	0.021
88	48	1057.03	1057	0.26±0.08	0.268±0.100	0.411	0.022
90	50	2186.274	2186.24 2	0.061±0.004	0.039±0.002	0.284	0.009
92	52	934.49	934.49	0.083±0.006	0.079±0.006	0.529	0.023
94	54	918.75	918.74	0.066±0.014	0.056±0.006	0.680	0.033
96	56	1750.498	1750.42	0.055±0.022	0.055	0.830	0.463
98	58	1222.93	1222	—	>0.0005±0.0000 2	0.974	1.215
100	60	212.530	212.531	1.11±0.06	1.210±0.044	1.096	1.535
102	62	151.78	151.75	1.66±0.34	1.842±0.241	1.164	1.622

Table (4): The calculated reduced transition probabilities B (E2) e<sup>2</sup>b<sup>2</sup> ↑ values are compared with that of experimental and, theoretical predications for <sup>40</sup>Zr nuclides.

Figure (3): Comparison between the B (E2) ↑ values of the present work for <sup>38</sup>Sr nuclei

with of ref.[5]experimental and other theoretical results.

Figure (4): Comparison between the B (E2) ↑ values of the present work for <sup>40</sup>Zr nuclei with of ref[5] experimental and other theoretical results.

### Discussion

Magic and closed shell nuclei have very few excited states at low excitation energy, their low transition probabilities include low collective motion for nucleons and indicated the validity of single –particle shell model ref.[6].

Therefore from tables (1&2) it is clear the experimental values for partial gamma widths Γ(E2) are greater than that estimated by Wiesskopf Γ(E2)<sub>W.u</sub> especially when the nucleon number deviated more and more from the magic neutron number. Since the cooperative effects appear between nucleons.

In figs. (1&2); <sup>38</sup>Sr and <sup>40</sup>Zr curves pass through minimum in the stable nuclei region of magic nuclide with N =50 and its neighboring', which are indicating allow collectivity of the excited states and the dominance of single –particle excited states. The comparison of the present values of B (E2) e<sup>2</sup>b<sup>2</sup>↑ with those reported in ref.[5] of experimental ,SSANM and



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FRDM can be explained by Figs (3&4) as that ; the present B (E2)  $e^2 b^2 \uparrow$  curves are in excellent agreement of that of ref .[5] experimental curves while B (E2)  $e^2 b^2 \uparrow$  curves predicted by SSANM have minimum values at magic neutron number N=50 and then are increasing gradually when nucleon number deviated more and more from N=50.

In FRDM B (E2)  $e^2 b^2 \uparrow$  curves have stable neutron number ranging between 42 and 56 the predicted values for the B (E2)  $e^2 b^2 \uparrow$  transition via N=42-54 are inaccurate relative to all B (E2)  $e^2 b^2 \uparrow$  values for the same transitions given in tables(3&4),also high value for B (E2)  $e^2 b^2 \uparrow$  at N=56 predicated by FRDM are unacceptable ,since N=56 is near neutron closed shell and each of Sr & Zr nuclei is with a proton sub-shell closure at 38 & 40 respectively this is leading to ignored residual interaction and considering the motion of nucleon is practically independent of that any other nucleon.

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