Some Results on a *-Generalized derivation and Jordan *-Generalized derivation

A.A.ALTAY

Department of mathematics, college of science, University of Baghdad, IRAQ

Mail: *ali_abd335 @yahoo.com*

بعض النتائج حول مشتقات المعممة-* ومشتقات جوردن المعممة-*

علي عبد عبيد الطائي

كلية العلوم – جامعة بغداد – العراق

Abstract

Let R be a *-ring, an additive mapping g: R → R is said to be a *-generalized derivation if there exists a *-derivation d: R → R such that $g(xy) = g(x)y^* + xd(y)$ for all $x,y \in R$. And an additive mapping g: R → R is said to be a Jordan *-generalized derivation if there exists a Jordan *-derivation d: R → R such that $g(x^2) = g(x)x^* + xd(x)$ for all $x \in R$. The purpose of this paper is to prove some result on a *-generalized derivation and Jordan*-generalized derivation which study some properties of these mappings, also we will give a relations between this mappings and commutatively or normality of *-ring R.

الخلاصـــة

لتكن R حلقة-*, تدعى الدالة التجميعية g: R→R مشتقات المعممة-* إذا حققت الشرط الأتي : لكل x,y في R فان $g(xy) = g(x)y^* + xd(y)$ حيث d دالة مشتقة-* وتسمى مشتقات جوردن المعممة-* إذا حققت الشرط الأتي : لكل x في R فان $g(x^2) = g(x)x^* + xd(x)$ حيث d دالة جوردن مشتقة-* في هذا البحث سوف ندرس بعض من خواص هذه الدوال التجميعية وعلاقتها في الحلقات-* الأبدالية والسوية.

1. Introduction

Throughout, R will represent an associative ring with center Z(R). A ring R is *n*-torsion free, if $nx = 0, x \in R$ implies $x = 0$, where *n* is a positive integer. Recall that R is prime if $aRb = (0)$ implies $a = 0$ or $b = 0$, and semiprime if $aRa = (0)$ implies $a = 0$. An additive mapping $x \to x^*$ on a ring R is called an involution if $(xy)^* = y^* x^*$ and $(x)^{**} = x$ for all $x, y \in R$. A ring equipped with an involution is called *-ring (see [1]). As usual the commutator $xy - yx$ will be

denoted by $[x, y]$. We shall use basic commutator identities $[xy, z] = [x, z]y + x[y, z]$ and $[x, yz] = [x, y]z + y[x, z]$ for all $x,y,z \in$R, (see [1, P.2]). Also we write $x o\, y=xy+yx$ for all $x, y \in$R (see [1]). An element x in a *-ring R is said to be hermitian if $x^* = x$ and skew-hermitian if $x^* = -x$. The sets of all hermitian and skew-hermitian elements of R will be denoted by H(R) and S(R), respectively. If R is 2-torsion free then every $x \in$ R can be uniquely represented in the form $2x = h + k$ where $h \in$ H(R) and $k \in$ S(R). An element $x \in$ R is called normal element if $xx^* = x^*x$, and if all the elements of R are normal then R is called a normal ring, an example is the ring of quaternion. A description of normal rings can be found in [2]. An additive mapping d: R$\rightarrow$R is called a derivation if $d(xy) = d(x)y + xd(y)$ holds for all pairs $x,y\in$R, and is called a Jordan derivation in case $d(x^2) = d(x)x + xd(x)$ is fulfilled for all $x \in$ R. Every derivation is a Jordan derivation. The converse is in general not true. A classical result of Herstein [3] asserts that every Jordan derivation on a prime ring of characteristic different from 2 is a derivation. Cusack [4] generalized Herstein's theorem to 2-torsion free semiprime ring . An additive mapping d: R$\rightarrow$R is called a *-derivation if $d(xy) = d(x)y^* + xd(y)$ holds for all $x,y\in$R, and is called a Jordan *-derivation in case $d(x^2) = d(x)x^* + xd(x)$ is fulfilled for all $x \in$ R, the concepts of *-derivation and Jordan*-derivation were first mentioned in [5] for more details see also ([6] and [7]]). A left (right) centralizer of R is an additive mapping T: R$\rightarrow$R which satisfies $T(xy) = T(x)y$ ($T(xy) = xT(y)$) for all $x, y \in$ R. A centralizer of R is an additive mapping which is both left and right centralizer. A left (right) Jordan centralizer of R is an additive mapping T: R$\rightarrow$R which satisfies $T(x^2) = T(x)x$ ($T(x^2)= xT(x)$) for all $x \in$ R. A Jordan centralizer of R is an additive mapping which is both left and right Jordan centralizer (see [8,9,10, and 11]). Every centralizer is a Jordan centralizer. B. Zalar [11] proved the converse when R is 2- torsion free semiprime ring. Inspired by the above definition Majeed and Altay[12] defines. A left (right) *-centralizer of R is an additive mapping T: R$\rightarrow$R which satisfies $T(xy) = T(x)y^*$ ($T(xy) = x^*T(y)$) for all $x, y \in$ R. A *-centralizer of R is an additive mapping which is both left and right *-centralizer. A left (right) reverse *-centralizer of a *-ring R is an additive mapping T: R$\rightarrow$R which satisfies $T(yx)=T(x)y^*$ ($T(yx) =x^*T(y)$) for all $x,y\in$R. A reverse *-centralizer of R is an additive mapping which is both left and right reverse *-centralizer. A left (right) Jordan *-centralizer of R is an additive mapping T: R$\rightarrow$R which satisfies $T(x^2)=T(x)\, x^*$ ($T(x^2)= x^*T(x)$) for all $x \in$ R. A Jordan *-centralizer of R is an additive mapping which is both left and right Jordan *-centralizer. Every reverse *-centralizer is a Jordan *-centralizer. Majeed and Altay [12] proved the converse when R is 2- torsion free semiprime *-ring. In [13] B. Hvala has defined the notion of generalized derivation as follows: An additive mapping g : R $\rightarrow$ R is said to be a generalized derivation if there exists a derivation d : R $\rightarrow$ R such that $g(xy) = g(x)y + xd(y)$ for all $x,y\in$R. Also, he called the maps of the form $x \rightarrow ax+xb$ where a, b are fixed elements in R by the inner generalized derivations. By a similar fashion to the definition of the *-derivation and the Jordan *-derivation,[14] M. N. Daif and M. S. Tammam, define the concepts of a *-generalized derivation and a Jordan *-generalized derivation as follows, an additive mapping g : R $\rightarrow$ R is said to be a *-generalized derivation if there exists a *-derivation d : R $\rightarrow$ R such that $g(xy) = g(x)y^* + xd(y)$ for all $x,y\in$R. And an additive mapping g : R $\rightarrow$ R is said to be a Jordan *-generalized derivation if

there exists a Jordan *-derivation d : R $\rightarrow$ R such that $g(x^2) = g(x)x^* + xd(x)$ for all $x \in$R. A generalized *-derivation g : R $\rightarrow$ R of *-ring R is called commuting mapping if $[g(x),x]=0$ for all $x \in R$. the second result give relation between normal *-ring and commuting generalized *-derivation. A Jordan generalized *-derivation g: R$\rightarrow$R is called inner if $g(x)=ax^*+xb$ for all $x \in$ R, and fixed element $a,b \in$ R.

2. The Main Results

In this note we will give some result on a *-generalized derivation and Jordan *-generalized derivation, and we will give a relation between a normal *-ring and commutative *-ring for prime and semiprime *-ring and *-generalized derivation and Jordan *-generalized derivation.

Since every *-generalized derivation and Jordan *-generalized derivation, but the converse in general is not true.

The following example illustrates this note:

Example2.1. Let R be a 2-torsion free semiprime *-ring and let $a \in$ R such that $[a,x]\neq 0$, for some $x \in$ R. Define a map g: R$\rightarrow$R as follows

$$g(x) = ax^*+xa \quad \text{for all } x \in \text{R}. \qquad (1)$$

For all $x \in$ R we have

$$g(x^2)=ax^{*2}+x^2a= ax^{*2}-xax^*+xax^*+x^2a$$

$$=(ax^*+xa)x^*+x(xa-ax^*) \quad \text{for all } x \in \text{R}. \qquad (2)$$

Also, define a map d: R$\rightarrow$R as follows

$$d(x)=xa-ax^* \quad \text{for all } x \in \text{R}. \qquad (3)$$

Clearly that d is an additive mapping, and

$$d(x)x^*+xd(x)= (xa-ax^*)\, x^*+x\, (xa-ax^*)$$

$$=ax^{*2}+x^2a=d(x^2) \quad \text{for all } x \in \text{R}. \qquad (4)$$

Then d is a Jordan *-derivation, and $g(x^2)=g(x)x^*+xd(x)$ is a *-generalized Jordan derivation. But g not a *-generalized derivation. If we assume that g is a *-generalized derivation then for x,y in R we have

$$g(xy)=ay^*x^*+xya=g(x)y^*+xd(y)$$

$$=ax^*y^*+xay^*+xya-xay^* \quad \text{for all } x,y \in \text{R}. \qquad (5)$$

Therefore,

$$a\,[x^*,y^*]=0 \qquad \text{for all } x,y \in R. \tag{6}$$

Replace x by xr^* in the relation (6) hence

$$a\,r\,[x^*,y^*]=0 \qquad \text{for all } x,y,r \in R. \tag{7}$$

Left multiplication the relation (6) by $r\in$ R, we get

$$r\,a\,[x^*,y^*]=0 \qquad \text{for all } x,y,r \in R. \tag{8}$$

Then from (7) and (8) one can obtain

$$[a,r]\,[x^*,y^*]=0 \qquad \text{for all } x,y,r \in R. \tag{9}$$

Replace y by r^* and x by $(za)^*$ in the relation (9) we get

$$[a,r]\,z\,[a,r]=0 \qquad \text{for all } x,y,r \in R. \tag{10}$$

Since R is a semiprime *-ring, then from relation (10) we obtain

$$[a,r]=0 \qquad \text{for all } r \in R. \tag{11}$$

This is contradiction, hence g is not *-generalized derivation.

Now we will give a result which is very important to obtain a *-generalized Jordan derivation on a 2-torstion free semiprime *-ring with an identity element.

Theorem2.2. Let R be a 2-torsion free *-ring with an identity element, g: R $\rightarrow$ R be an additive mapping satisfying the relation

$$g(x^3)=g(x)x^{*2}+xd(x)x^*+x^2d(x) \qquad \text{for all } x \in R. \tag{12}$$

Where d is a Jordan *- derivation. Then g is a *-generalized Jordan derivation.

For the proof of the above theorem we shall need the lemma below.

Lemma2.3. Let R is a *-ring with one then:

1- The mapping g: R $\rightarrow$ R be a Jordan *-generalized derivation ($g(x^2) = g(x)x^* + xd(x)$) for all $x\in$R.) for some Jordan *-derivation d : R $\rightarrow$ R if and only if $g(y)=ay^*+d(y)$ for all $y\in$R, where d is a Jordan*-derivation.

2- If The mapping g: R $\rightarrow$ R be a *-generalized derivation ($g(xy) = g(x)y^* + xd(y)$) for all $x,y\in$R.) for some *-derivation d: R $\rightarrow$ R then, $g(x) =ax^*+d(x)$ for all $x\in$R.

Proof: (1) We have

$$g(x^2)=g(x)x^*+xd(x) \qquad \text{for all } x \in R. \tag{13}$$

Linearization (13) we get

$$g(xy+yx)=g(x)y^*+xd(y)+g(y)x^*+yd(x) \quad \text{for all } x,y \in R. \tag{14}$$

Replace y by $xy+yx$ in (14) and since R is 2-torsion free we get

$$g(xyx)= g(x)y^*x^*+xd(y)x^*+xyd(x) \qquad \text{for all } x,y \in R. \tag{15}$$

Replace x=1, in (15) and since d(1)=0 we get

$$g(y)=ay^*+d(y) \quad \text{for all } y\in R. \tag{16}$$

Where a=g(1). Conversely, replace y by y^2 in the relation (16) we obtain

$$g(y^2)=ay^{*2}+d(y^2)=(ay^*)y^*+d(y)y^*+yd(y)$$
$$=g(y)y^*+yd(y) \quad \text{for all } y\in R. \tag{17}$$

Proof (2) we have

$$g(xy) = g(x)y^* + xd(y) \quad \text{for all } x,y\in R \tag{18}$$

Replace y=1,in the relation (18) we get

$$g(x)=ax^*+d(x) \quad \text{for all } x\in R \tag{19}$$

Where a=g(1).

<u>**Proof of theorem2.2:**</u>

Linearization the relation (12), we get

$$g(x^2y+yx^2+xy^2+y^2x+xyx+yxy)=g(x)x^*y^*+g(x)y^*x^*+g(x)y^{*2}+$$
$$g(y)x^*y^*+g(y)y^*x^*+g(y)x^{*2}+xd(x)y^*+xd(y)x^*+yd(x)x^*+yd(y)x^*$$
$$+xd(y)y^*+yd(x)y^*+xyd(x)+yxd(x)+y^2d(x)+xyd(y)+yxd(y)+x^2d(y)$$
$$\text{for all } x,y \in R. \tag{20}$$

Replace x by $-x$ in the relation (20) we obtain

$$g(x^2y+yx^2+xy^2+y^2x+xyx+yxy)=g(x)x^*y^*+g(x)y^*x^*-g(x)y^{*2}-$$
$$g(y)x^*y^*-g(y)y^*x^*+g(y)x^{*2}+xd(x)y^*+xd(y)x^*+yd(x)x^*-yd(y)x^*$$
$$-xd(y)y^*-yd(x)y^*+xyd(x)+yxd(x)-y^2d(x)-xyd(y)-yxd(y)+x^2d(y)$$
$$\text{for all } x,y \in R. \tag{21}$$

According to the relation (20), (21) we obtain

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$$g(x^2y+yx^2+xyx)=g(x)x^*y^*+g(x)y^*x^*+g(y)x^{*2}+xd(x)y^*$$
$$+yd(x)x^*+xd(y)x^*+xyd(x)+yxd(x)+x^2d(y)$$
$$\text{for all } x,y \in R. \quad (22)$$

Now replace x by 1 in the relation (22) and since R be a 2-torsion free and d(1)=0 , we get, $g(y)=ay^*+d(y)$ for all $x \in R$, then by using Lemma2.3 we get g is a *-generalize Jordan derivation.

M. N. Daif and M. S. Tammam [6], prove that: let R be a 6-torsion free semiprime *-ring and let g: $R \rightarrow R$ be an additive mapping satisfying the relation

$$g(xyx)=g(x)y^*x^*+xd(y)x^*+xyd(x) \quad \text{for all } x,y \in R. \quad (23)$$

For a Jordan *- derivation d of R. Then g is a Jordan *-generalized derivation. In this theorem we will prove same result to a M. N. Daif and M. S. Tammam, but in case R is a 2-torsion free semiprime *-ring.

Thorem2.4: let R be a 2-torsion free semiprime *-ring and let g: $R \rightarrow R$ be an additive mapping satisfying the relation (23), for a Jordan *- derivation d of R. Then g is a Jordan *-generalized derivation.

To prove above theorem we shall need the following result:

Lemma2.5. Let R be a *-ring, then

1- The mapping g: $R \rightarrow R$ be a Jordan *-generalized derivation ($g(x^2) = g(x)x^* + xd(x)$ for all $x \in R$.) for some Jordan *-derivation $d : R \rightarrow R$ if and only if g=T+d where T is a left Jordan *-centralizer and d is a Jordan *-derivation.

2-The mapping g: $R \rightarrow R$ be a *-generalized derivation ($g(xy) = g(x)y^* + xd(y)$ for all $x,y \in R$.) for some *-derivation $d : R \rightarrow R$ if and only if g=T+d where T is a left *-centralizer and d is a *-derivation.

Proof: (1) We have g be a Jordan *-generalized derivation

Let T=g-d clear that g=T+d, since g and d is additive then T is additive,

Also,
$$T(x^2)= g(x^2)-d(x^2)= g(x)x^* + xd(x)-d(x)x^* -xd(x)$$
$$=(g-d)(x)\, x^*=T(x)x^* \quad \text{for all } x \in R. \quad (24)$$

Hence T is a left Jordan *-centralizer, and therefore g=T+d, T is a left Jordan *-centralizer and d is a Jordan *-derivation.

Conversely,

We have, g=T+d where T is a left Jordan *-centralizer and d is a Jordan *-derivation, then
$$g(x^2)=T(x^2)+d(x^2)=T(x)x^*+d(x)x^*+xd(x)$$
$$=(T+d)(x)x^*+xd(x)=g(x)x^*+xd(x) \quad \text{for all } x \in R. \quad (25)$$

Hence g is a Jordan *-generalized derivation.

Proof: (2) We have g be a *-generalized derivation

Let T=g-d clear that g=T+d, since g and d is additive then T is additive,

Also,

$$T(xy)= g(xy)-d(xy)= g(x)y^* + xd(y)-d(x)y^* -xd(y)$$

$=(g\text{-}d)(x)\, y^*=T(x)y^*$ for all $x,y\in R$. (26)

Hence T is a left *-centralizer, therefore g=T+d T is a *-centralizer and d is a *-derivation conversely,

We have, g=T+d where T is a left *-centralizer and d is a *-derivation, then

$$g(xy)=T(xy)+d(xy)=T(x)y^*+d(x)y^*+xd(y)$$
$$=(T+d)(x)y^*+xd(y)=g(x)y^*+xd(y) \quad \text{for all } x,y\in R. \qquad (27)$$

Hence g is a *-generalized derivation.

Proof of theorem2.4:

If R have an identity element, put x=1 in (23), and using Lemma2.3 we get g is a Jordan *-generalized derivation, so we define the mapping T: R→R by $T(x)=(g\text{-}d)(x)$ for all $x\in R$, since g and d is additive then T is additive, also

$$T(xyx)=g(xyx)-d(xyx)= g(x)y^*x^*+xd(y)x^*+xyd(x)-d(x)y^*x^*-xd(y)x^*-xyd(x)$$
$$=(g\text{-}d)(x)y^*x^*=T(x)y^*x^* \quad \text{for all } x,y\in R. \qquad (28)$$

Hence same away for prove of [Theorem (1.1.35) in [12]] we get T is a left reverse *-centralizer and therefore T is a left Jordan *-centralizer, and therefore g=T+d where T is a left Jordan *-centralizer and d is Jordan *-derivation, therefore by using Lemma2.5 we g is a Jordan *-generalized derivation.

In this theorem we will give a relation between generalized *-derivation and commutative *-ring.

Proposition2.6. Let R be a semiprime *-ring, and let g: R→R be a nonzero *-generalized derivation, then $g(x) \in Z(R)$ for all $x\in R$, and if R is a prime *-ring then R is commutative *-ring.

To prove this result we need the following lemma:

Lemma 2.7.[48]. Let R be a semiprime ring, and let a be an element in R , if $a[x,y] = 0$ for all $x, y \in R$, then there exists an ideal U of R such that $a \in U \subset Z(R)$.

Proof of proposition2.6:

Replace y by yz in (18) we get

$$g(x(yz))=g(x)z^*y^*+xd(y)z^*+xyd(z), \quad \text{for all } x,y,z \in R, \qquad (29)$$

Also,

$$g((xy)z) =g(x)y^*z^*+xd(y)z^*+xyd(z), \quad \text{for all } x,y,z \in R, \qquad (30)$$

If we comporting (29) and (30) we obtain

$$g(x)\, [y^*,z^*]=0 \quad \text{for all } x,y,z \in R, \qquad (31)$$

By using Lemma2.7 we get $g(x) \subseteq Z(R)$ for all $x \in R$, if R is prime *-ring from (31) we get $g(x)\, r\, [y^*,z^*]=0$ for all $x,y,z \in R$, since R is a prime *-ring and $0 \neq g$, we get R is commutative *-ring.

M. Breŝar and J. Vukman In [16] proved that, if R be a non-commutative prime *-ring of characteristic different from2, then R is normal ring if and only if there exists a nonzero commuting Jordan *-derivation. In this theorem, we will give a result similar to the result of M. Breŝar and J. Vukman [16], but in case Jordan generalized*-derivation.

Theorem2.8. Let R be a 2-torsion free non-commutative prime *-ring. Then R is normal *-ring if, and only if, there exists a non-zero commuting Jordan *-generalized derivation.

To prove this theorem we need the following results:

Lemma 2.9.[12]. Let R be a 2-torsion free prime *-ring, I an ideal of R, and T: R→R an additive mapping which satisfies $T(x^2) = T(x)\, x^*$ for all $x \in I$, then R is commutative.

Proof of theorem2.8:

Define the mapping g: R→R by

$$g(x)=x+x^* \qquad \text{for all } x \in R. \tag{32}$$

Clearly that g is additive, also

$$g(x^2)=x^{*2}+x^2= x^{*2}+xx^*-xx^*+ x^2=g(x)x^*+x(x-x^*) \quad \text{for all } x \in R. \tag{33}$$

Define a map d: R→R as follows

$$d(x)=x-x^* \qquad \text{for all } x \in R. \tag{34}$$

Clearly that d is additive, also

$$d(x^2)=x^2-x^{*2}= x^2-xx^*+xx^*- x^{*2}=(x-x^*)x^*+x(x-x^*)$$

$$=d(x)x^*+xd(x) \qquad \text{for all } x \in R. \tag{35}$$

We get d is a Jordan *-derivation, hence from (34) and (35) we get $g(x^2)=g(x)x^*+xd(x)$ for all $x \in R$, is a Jordan generalized *-derivation. Also since R is a normal *-ring we obtain

$$[g(x),x]=[x+x^*,x]=[x,x]+[x^*,x]=0 \qquad \text{for all } x \in R. \tag{36}$$

Hence g is commuting mapping, also if g=0, we get

$$x+x^*=0 \qquad \text{for all } x \in R. \tag{37}$$

Replace x by xy in the relation (37) and using (37) we get

$$xy+yx=0 \quad \text{for all } x,y \in R. \tag{38}$$

Since R is a 2-torsion free one can show that from above relation

$$x^2=0 \quad \text{for all } x\in R. \tag{39}$$

Hence left multiplication (38) by x and using (39) we get

$$x\,y\,x=0 \quad \text{for all } x,y\in R. \tag{40}$$

By semiprime of a *-ring R, we get $x=0$ for all $x\in R$, which is Contradiction, hence g be a non-zero commuting Jordan *-generalized derivation.

To show the converse, let $g : R \to R$ be a a nonzero commuting *-generalized Jordan derivation ($g(x^2) = g(x)x^* + xd(x)$ for all $x,y\in R$.) for some Jordan *-derivation $d : R \to R$ by Lemma2.5 we get, g=T+d where T is a left Jordan *-centralizer and d is a Jordan *-derivation, if $0\neq T$ then by Lemma2.9 we get R is commutative *-ring which is a contradiction, hence T=0, therefore $0\neq g=d$, hence by M. Brešar and J. Vukman [16] we get R is a normal *-ring.

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