On some result of topological projective modules

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Abstract

In this paper, we given and study the finite direct sum and tensor product of topological projective modules. We obtain some results and properties to relate the direct sum and tensor product of topological projective modules.

Keyword: tensor product, topological rings, topological modules, topological projective modules

بعض النتائج حول المقاسات الاسقاطية التبولوجية

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المستخلص

في هذا البحث تم اعطاء ودراسة الجمع المباشر المنتهي والضرب التنسوري للمقاسات الأسقاطية التبولوجية . حصلنا على بعض النتائج والخواص التي تربط بين الجمع المباشر والضرب التنسوري للمقاسات الاسقاطية التبولوجية.

الكلمات المفتاحية: الضرب التنسوري , الحلقات التبولوجية, المقاسات التبولوجية, المقاسات الاسقاطية التبولوجية.

Introduction

Recentaly C.Nilson introduce and discussion new concept of topological is said to be topological modules[8]. After that others anothers modifaction these space like,M.Mahuoub in 2002,he introduced and studied some type of topological modules spaces is called topological injective modules[3], also Al-Anbaki in 2005 given new properties of topological modules is said to be topological projective modules[1] ,but here we give more one than of these spaces and we study and discussion more properties of that means.we generalized some definition and theorem appear in[2] and in this work, we given relationships between the direct sum and tensor product of topological projective modules to obtain some properties of these characterization such as direct sum and tensor product.

In this we will study topological modules, topological projective modules and the introducing some results in tensor product of topological projective modules.

Some basic cocept of topological modules

In this section we give the fundamental concepts releated to this:

Definition [2]

A topological group is a set G together with two structures:

1-G is a group.

2-Topology on G.

The two structures are compatible: i.e.,

The group (binary operation) $\mathcal{M}: G \times G \longrightarrow G$

And the inversion $\mathcal{V}: G \xrightarrow{\sim} G$ are both continuous map.

Example [2]

Every group is a topological group with discrete topology G =R

On the usuall topology is topological group.

Definition [4]

The topological ring R is a non-empty set together with two structures:

1-A map $(x,y) \longrightarrow x+y$ from $R\times R \longrightarrow R$ be continuous.

2-A map $X \longrightarrow -X$ from $R \longrightarrow R$ be continuous.

3-A map $(x,y) \longrightarrow xy$ from $R\times R \longrightarrow R$ be continuous.

Example [7]

The discrete topology on a ring R is topological ring.

The usual topology on R is topological ring.

Definition [8]:

Let R be topological ring. The set P is called left topological module on R if

1-P is left module on R.

2-P is topological group.

3-A map $(\lambda,x):R\times M \longrightarrow M$ defined by $(\lambda,x) = \lambda x$, $\lambda \in R, x \in M$

On the same way we definition the right topological module.

Example [7]:

1-The module on a ring from topological module with discrete topology.

2-Every abelian topological group is topological module on the ring Z.

Definition [5]:

Let P,P' be two topological modules on the topoplogical ring R, then

f : p $\longrightarrow$ p' is called topological module homomorphism if:

1-f is module homomorphism .

2-f is continuous map.

Definition [3]:

Let p be topological module of R (topological ring), the subset M of R be topological

submodule of p:

1-M be submodule p.

2-M be topological subgroup of p.

3-A map $(\lambda, x) \longrightarrow \lambda x$ from is continuous $R \times M \longrightarrow M$

Topological projective modules

In this section, we give some basic concepts of topological projective modules and the direct sum of topological projective modules.

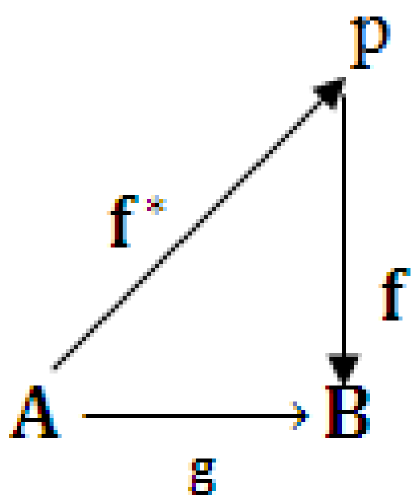
Definition [1]:

A topological module p is called topological projective module if for all topological module epimorphism and for all $g: A \longrightarrow B$

topological module morphism $f: p \longrightarrow B$, there exists a

topological module morphism $f^*: p \longrightarrow A$, for which the

following diagram commutes:



Notation [3]:

Ker f is a topological submodule of p where f is a topological module homomorphism from p into p'.

Proposition [6]:

If p be topological projective on R and p be discrete topological module, then P is topological rojective module .

Definition [4]:

Let M be topological submodule of topological module E, M is called topological discrete sum of E if there exists another submodule N of E such that E is topological direct sum of M and N thus $E \cong N \oplus M$.

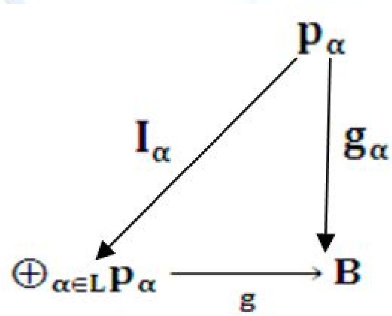
Theorem [5]:

Let $\{p_\alpha\}_{\alpha \in L}$ be family of topological modules on topological ring R,B be

Topological module homomorphism $g_\alpha : p_\alpha \longrightarrow B$ for all $\alpha \subset L$ there

exsists a unique topological module homomorphism $g: \oplus_{\alpha \in L} p_\alpha \longrightarrow B$

for which the following diagram commutes:



Theorem [6]:

Let $\{p_\alpha\}_{\alpha \in L}$ be finite family of topological module of topological ring R

then the topological direct sum $p = \bigoplus_{\alpha \in L} p_\alpha$ be topological projective

module if p_α is topological module.

The Tensor product of topological projective modules:

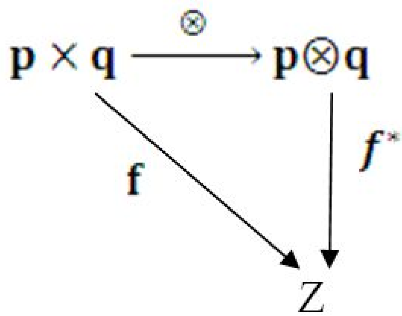
Definition [6]:

Let P and q be topological projective module, let $f: p \longrightarrow M$ and $g: q \longrightarrow N$ be topological module morphism where M and N be topological module then there exists a unique topological module morphism from to $p \otimes q$ denoted by $p \otimes q$ such that $f \otimes g$

$(f \otimes g)(p \otimes q) = (f(p) \otimes (g(q))$, for all $p \in P \ \forall\ q \in q$

Definition [7]:

Let P, q be two topological modules of the ring R thus the tensor product over R, $p \otimes q$ is an abelian group together with abilinear map $\otimes p \times q \longrightarrow p \otimes q$ which is universal for every abelian group Z and there exsist a unique homomorphisms $f^*: p \otimes q \longrightarrow Z \ni f^* \circ \otimes = f$



<u>Theorem:</u>

Let $\{p_\alpha\}_{\alpha \in L}$ be finite family of topological module of topological ring R then

the topological tensor product $p = \otimes_{\alpha \in L} p_\alpha$ be topological projective module iff

p_α is topological module.

<u>Proof :</u>

Let p_α be topological module there exsists topological module

homomorphism $f^*_\alpha : P_\alpha \longrightarrow A$

$g \circ f^*_\alpha \longrightarrow f \circ I_\alpha$

$\exists!\, h : P \longrightarrow A$

$h \circ I_\alpha = g \circ f^*_\alpha$

$= g \circ (h \circ I_\alpha)$

$= (g \circ h) \circ I_\alpha \quad \forall\, \alpha \in L$

$f = g \circ h$

P is topological projective module f_α

Let A,B be topological modules on R an f^* , be topological module homomorphism and g onto (too dule) thus P_α be topological module.

Theorem :

Let $\{P_i\}_{1 \le i \le n}$ be finite topological module of topological ring R then the tensor

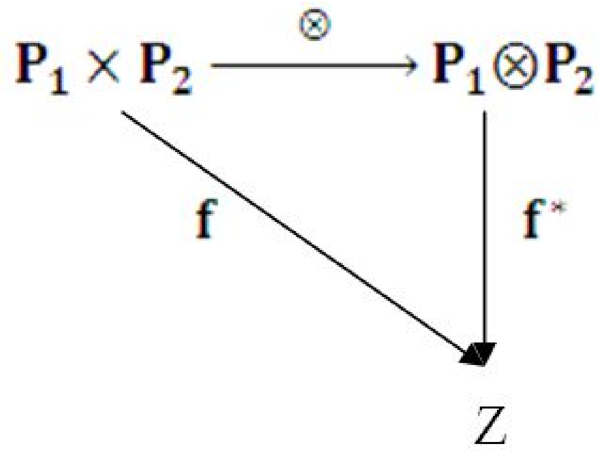
Product $\otimes_{i=1}^{n} P_i = P_1 \otimes P_2 \otimes P_3 \otimes \ldots \ldots \ldots \otimes P_n$ be topological projective module iff P_i be topological module.

Proof:

Let P_1 and P_2 be two topological module to show $P_1 \otimes P_2$ be topological projective module since by definition of tensor product

for every abelian group Z and every linear map $f: P_1 \otimes P_2 \longrightarrow Z$ there exsists a unique group homomorphism $f^*: P_1 \otimes P_2 \longrightarrow Z$ that such

$f^* \circ \otimes = f$



Thus $f^*{}_i, i = 1, 2 : P_i \longrightarrow A$

$j \circ f^*{}_i \longrightarrow f \circ f_i$

$\exists\ h : P \longrightarrow A$

$h \circ f_i = g \circ f^*{}_i \quad i = 1, 2$

$= g \circ (h \circ f_i)$

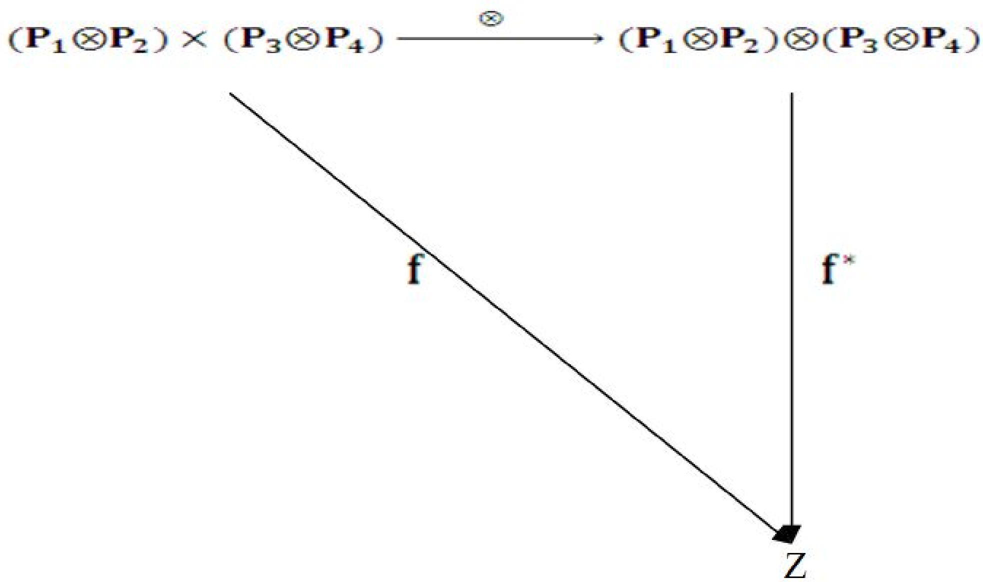
$\Longrightarrow f = g \circ h$

$\Longrightarrow P_1 \otimes P_2$ be topological projective module.

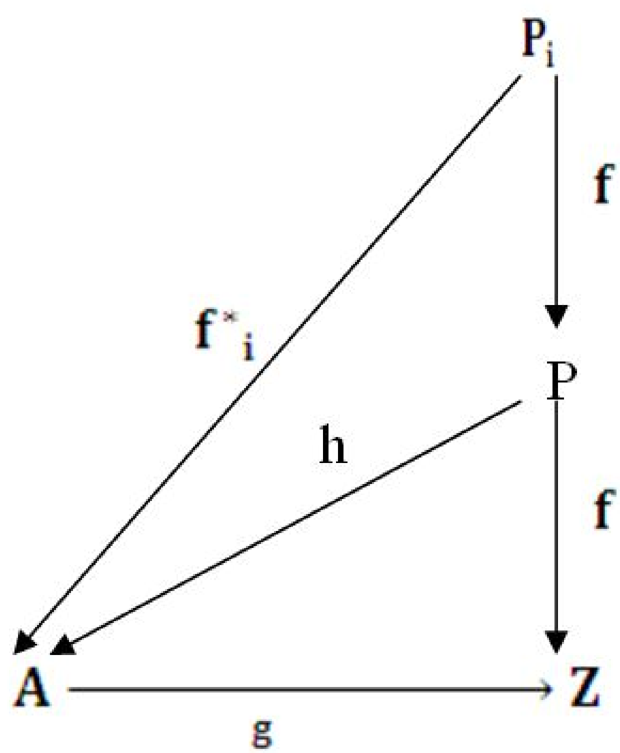
On the same way

$\otimes_{i=1}^{n} P_i$ Be topological projective modules by steps $(P_1 \otimes P_2) \otimes (P_3 \otimes P_4) \otimes (\dots\dots\dots) \otimes P_n$

Where



And



Where $P = \otimes_{i=1}^{n} P_i$ and in general $\otimes_{i=1}^{n} P_i$ be topological projective modules iff $P_i(i = 1, \ldots \ldots \ldots, n)$ be topological module.

Next, the following corollary it clear and we can get from above theorem.

corollary :

Let P_i, P_i' be topological projective modules for i=1,2,…….,n

1- $P_1 \otimes (\oplus_{i=2}^{n} P_i)$ be topological projective modules.

2- $(\otimes_{i=2}^{n} P_i) \oplus P_1$ be topological projective modules.

3- $(\otimes_{i=1}^{n} P_i) \oplus (\otimes_{i=1} P_i')$ be topological projective modules.

4- $(\oplus_{i=1}^{n} P_i) \otimes (\oplus_{i=1}^{n} P_i')$ be topological projective modules.

5- $(\oplus_{i=1}^{n} P_i) \otimes (\otimes_{i=1}^{n} P_i')$ be topological projective modules.

6- $(\otimes^{n} P_i) \otimes (\oplus_{i=1}^{n} P_i')$ be topological projective modules.

7- $(\otimes_{i=1}^{n} P_i) \otimes (\otimes_{i=1}^{n} P_i')$ be topological projective modules.

8- $(\otimes_{i=1}^{n} P_i) \oplus (\otimes_{i=1}^{n} P_i')$ be topological projective modules.

Proof :

3- since $\otimes_{i=1}^{n} P_i = P_1 \otimes P_2 \otimes P_3 \otimes \ldots \ldots \ldots \otimes P_n$ and be topological projective module and $\otimes_{i=1}^{n} P_i' = P_1 \otimes P_2 \otimes P_3 \otimes \ldots \ldots \ldots \otimes P_n$ be topological projective module

thus, $(\otimes_{i=1}^{n} P_i) \otimes (\otimes_{i=1}^{n} P_i')$ be topological projective module such that

$(\otimes_{i=1}^{n} P_i) \otimes (\otimes_{i=1}^{n} P_i') = P_1 \otimes P_2 \otimes P_3 \otimes \ldots \ldots \otimes P_n \otimes P_1' \otimes P_2' \otimes \ldots \ldots \otimes P_i'$

be topological projective modules that conclusion(4-2),(4-3) and (4-4).of theorem.

and be tpological projective modules

7- since $(\otimes_{i=1}^{n} P_i) = P_1 \otimes P_2 \otimes P_3 \otimes \ldots \ldots \ldots \otimes P_n$

and $\otimes_{i=1}^{n} P_i' = p_1' \otimes p_2' \otimes p_3' \otimes \ldots . \otimes p_n'$ be topological projective module

thus $({}_{i=1}^{n} \otimes P_i) \otimes ({}_{i=1}^{n} \otimes P_i') - P_1 \otimes P_2 \otimes P_3 \otimes \ldots \ldots \otimes P_n \otimes P_1' \otimes P_2' \otimes P_3' \otimes \ldots . \otimes P_n'$

be topological projective modules, that conclusion of theorem (4.2),(4.3) and (4.4)

on the some way we proof other properties.

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